AN INTERNAL-MULTIPLE *ELIMINATION* ALGORITHM FOR ALL REFLECTORS FOR 1D EARTH PART I: STRENGTHS AND LIMITATIONS

YANGLEI ZOU and ARTHUR B. WEGLEIN

M-OSRP, Physics Dept., University of Houston, 617 Science & Research Bldg. 1, Houston, TX 77004-5005, U.S.A. yzou6@uh.edu

(Received July 10, 2014; revised version accepted August 16, 2014)

ABSTRACT

Zou, Y. and Weglein, A.B., 2014. The internal-multiple *elimination* algorithm for all reflectors for 1D earth. Part I: strengths and limitations. *Journal of Seismic Exploration*, 23: 393-404.

The ISS (Inverse-Scattering-Series) internal-multiple attenuation algorithm can predict the correct time and approximate amplitude for all internal multiples without any subsurface information. In practice, an energy minimization adaptive subtraction step is often called upon to then remove the residual/attenuated internal multiple. However, the energy minimization criteria behind the adaptive subtraction algorithm can fail with interfering or proximal primary and multiple events. The latter can occur with complex off-shore plays and very often occurs with on-shore plays. In 2003, Weglein proposed a three-pronged strategy for providing an effective response to this pressing and prioritized challenge. One part of the strategy is to develop an internal-multiple elimination algorithm that can predict both the correct amplitude and correct time for all internal multiples. The ISS internal-multiple elimination algorithm for all first-order internal multiples generated from all reflectors in a 1D earth is proposed in part I of this paper. The primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of this new algorithm. In part II, we show that we can replace all b_1 in the elimination algorithm by $b_1 + b_3$ to mitigate this limitation. Moreover, this elimination algorithm based on the ISS internal-multiple attenuation algorithm is derived by using reverse engineering to provide the difference between eliminate and attenuate for a 1D earth. This particular elimination algorithm is model type dependent since the reverse engineering method depends on the specific relationship between reflection and transmission coefficients for an acoustic medium. The ISS internal-multiple attenuation algorithm is model type independent. Future work will pursue the development of an eliminator for a multi-dimensional earth by identifying terms in the inverse scattering series that have that purpose and capability.

KEY WORDS: internal multiple elimination, inverse scattering subseries, adoptive subtraction, internal multiple attenuation.

0963-0651/14/$5.00

INTRODUCTION

The inverse-scattering-series allows all seismic processing objectives, such as free-surface-multiple removal and internal-multiple removal to be achieved directly in terms of data, without any estimation of the earth's properties. For internal-multiple removal, the ISS internal-multiple attenuation algorithm can predict the correct time and approximate and well-understood amplitude for all internal multiples generated from all reflectors without any subsurface information. If the events in the data are isolated, the energy minimization adaptive subtraction can fill the gap between the attenuation algorithm and elimination. However, under certain circumstances, events often interfere with each other in both on-shore and off-shore exploration plays. In these cases, the criteria of energy minimization adaptive subtraction may fail and completely removing internal multiples (without damaging proximal primaries) becomes more challenging and beyond the current capability of the petroleum industry.

For dealing with this challenging problem, Weglein et al. (2003) proposed a three-pronged strategy including:

(1) Develop the ISS prerequisites for predicting the reference wave field and to produce de-ghosted data for on-shore applications (Wu and Weglein, 2014a,b).

(2) Develop internal-multiple elimination algorithms from ISS.

(3) Develop a replacement for the energy-minimization criteria for adaptive subtraction that always align with and support items (1) and (2).

For the second part of the strategy, that is, to upgrade the ISS internal-multiple attenuator to eliminator, the strengths and limitations of the ISS internal-multiple attenuator are noted and reviewed. The ISS internal-multiple attenuator always attenuates all first-order internal multiples from all reflectors at once, automatically and without subsurface information. That is a tremendous strength, and is a constant and holds independent of the circumstances and complexity of the geology and the play. The primaries in the reflection data that enters the algorithm provides that delivery, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, when they enter the ISS internal-multiple algorithm will alter the higher order internal multiples and thereby assist and cooperate with higher order ISS internal-multiple attenuation terms, to attenuate higher order internal multiples. However, there is a downside, a limitation. There are cases when internal multiples that enter the attenuator can predict spurious events. That is a well-understood shortcoming of the leading order term, when taken in isolation, but is not an issue for the entire ISS internal-multiple capability. It is anticipated by the ISS and higher order ISS internal multiple terms exist to precisely remove that issue of spurious event prediction, and when taken together with the first-order term, no longer experiences spurious event prediction. Ma et al. (2012), Ma and Weglein (2014) and Liang et al. (2013, 2014), provided those higher order terms for internal multiple attenuation and for spurious events removal. In a similar way, there are higher order ISS internal multiple terms that provide the elimination of internal multiples when taken together with the leading order attenuator term. There are early discussions in Ramírez (2007). Herrera and Weglein (2012) derived an algorithm that can eliminate all first-order internal multiples generated at the shallowest reflector for a normal incident plane wave on a 1D earth. Part I of this paper proposes a general elimination algorithm for all first-order internal-multiples generated from all reflectors in a 1D earth. The primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without requiring the primaries to be identified or in any way separated. The other events in the input reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of the current algorithm. In part II (Zou and Weglein, 2014), b_1 in the elimination algorithm is replaced by $b_1 + b_3$ to mitigate this limitation. Moreover, this elimination algorithm begins with the ISS internal-multiple attenuation algorithm, and is derived by using reverse engineering method. It is model type dependent since the reverse engineering and thereby providing the difference between elimination and attenuation is model type dependent. The ISS internal-multiple attenuation algorithm is model type independent.

ISS INTERNAL-MULTIPLE ATTENUATION ALGORITHM AND ATTENUATION FACTOR (1D NORMAL INCIDENCE)

First, we provide a review of the ISS internal-multiple attenuation algorithm before we introduce the internal-multiple elimination algorithm. The ISS internal-multiple attenuation algorithm is first given by Araújo et al. (1994) and Weglein et al. (1997). The 1D normal incidence version of the algorithm is presented as follows:

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \quad , \tag{1}$$

where $b_1(z)$ which is closely related to the data is the water speed migration of the data due to a 1D normal incidence spike plane wave. ε_1 and ε_2 are two small positive number introduced to avoid self interaction. $b_3^{IM}(k)$ is the predicted internal multiples in the vertical wavenumber domain. This equation can predict the correct time and approximate amplitude of all first-order internal multiples.

The procedure of predicting a first-order internal multiple generated at the shallowest reflector is shown in Fig. 1. The ISS internal-multiple attenuation algorithm uses three primaries in the data to predict a first-order internal multiple (note that this algorithm is model type independent and it takes account all possible combinations of primaries that can predict internal multiples. These figures are included to provide an intuitive understanding of the algorithm). From the figure we can see, every sub event on the left hand side experiences several phenomena making its way down to the earth then back to the receiver. When compared with the actual internal multiple on the right hand side, the events on the left hand side have extra transmission coefficients as shown in red. Multiplying all those extra transmission coefficients, we get the attenuation factor $T_{01}T_{10}$ for this first-order internal multiples having their single downward reflection at the shallowest reflector. All first-order internal multiples generated at the shallowest reflector have the same attenuation factor.

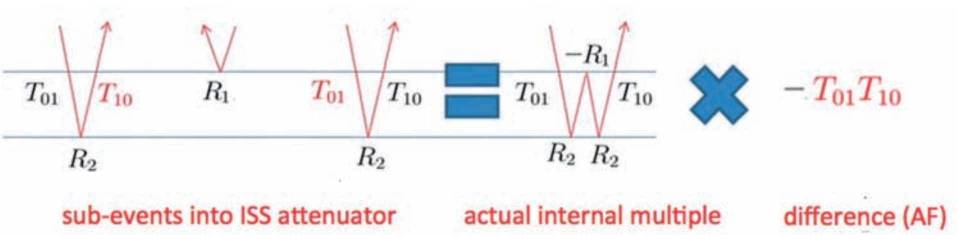

Fig. 1. An example of the Attenuation Factor of a first-order internal multiple generated at the shallowest reflector, notice that all red terms are extra transmission coefficients.

Fig. 2 shows the procedure of predicting a first-order internal multiple generated at the next shallowest reflector. In this example, the attenuation factor is $(T_{01}T_{10})^2(T_{12}T_{21})$.

The attenuation factor is the difference between an internal multiple and its ISS predicted attenuator.

The attenuation factor, AF_j, in the prediction of internal multiples is given by the following:

$$AF_j = \begin{cases} T_{0,1}T_{1,0} & (\text{for } j = 1) \\ \prod_{i=1}^{j-1}(T_{i-1,i}^2 T_{i,i-1}^2)T_{j,j-1}T_{j-1,j} & (\text{for } 1 < j < J) \end{cases} . \tag{2}$$

The attenuation factor AF_j can also be expressed in terms of reflection coefficients:

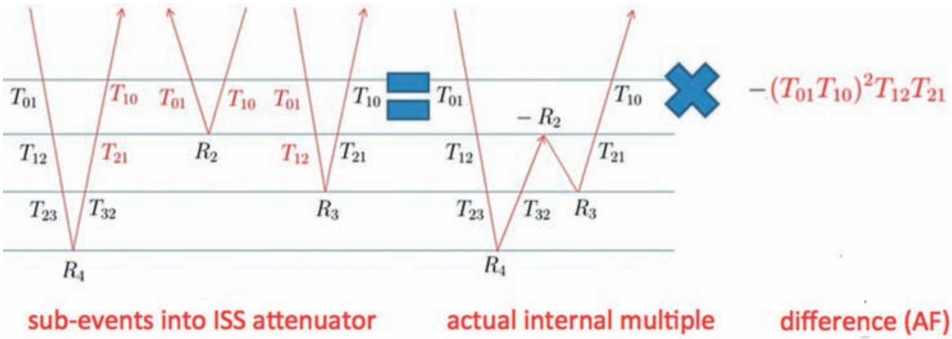

Fig. 2. An example of the Attenuation Factor of a first-order internal multiple generated at the next shallowest reflector, notice that all red terms are extra transmission coefficients.

$$F_j = \begin{cases} 1 - R_1^2 \quad (\text{for } j = 1) \\ (1 - R_1^2)^2(1 - R_2^2)^2 \ldots (1 - R_{j-1}^2)^2(1 - R_j^2) \quad (\text{for } 1<j<J) \end{cases} \quad . \tag{3}$$

The subscript j represents the generating reflector, and J is the total number of interfaces in the model. The interfaces are numbered starting with the shallowest reflector.

ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM (1D NORMAL INCIDENCE)

The discussion above demonstrates that all first-order internal multiples having their downward reflection at the same reflector have the same attenuation factor. We can see the attenuation factor contains all transmission coefficients from the shallowest reflector down to the reflector generating the multiple. And from the examples (shown in Figs. 1 and 2) we can see the middle event contains all the information about those transmission coefficients. Therefore, the idea is to modify the middle term in the attenuation algorithm to remove the attenuation factor and make the attenuation algorithm an eliminator. That is from the attenuator

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} b_1(z') \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \ , \tag{4}$$

to the eliminator

$$b_3^{IM}(k) = \int_{-\infty}^{\infty} dz e^{ikz} b_1(z) \int_{-\infty}^{z-\varepsilon_2} dz' e^{-ikz'} F[b_1(z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{ikz''} b_1(z'') \ . \tag{5}$$

By introducing a new function, g(z), for which the amplitude of each event corresponds to a reflection coefficient, we find a way to construct $F[b_1(z)]$ by using $b_1(z)$ and g(z). After that, we find an integral equation relating $b_1(z)$ and g(z). $F[b_1(z)]$ is determined (Zou and Weglein, 2014) as follows:

$$F[b_1(z)] = b_1(z)$$

$$/[1 - \{ \int_{z-\varepsilon}^{z+\varepsilon} dz'g(z')\}^2][1 - \int_{-\infty}^{z-\varepsilon} dz'b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz''g(z'')]^2 \ , \quad (6)$$

$$g(z) = b_1(z)/[1 - \int_{-\infty}^{z-\varepsilon} dz'b_1(z') \int_{z'-\varepsilon}^{z'+\varepsilon} dz''g(z'')] \ . \quad (7)$$

To derive $F[b_1(z)]$ from $b_1(z)$, g(z) must first be solved in eq. (7). Thereafter, g(z) is integrated into eq. (6). Now we will show one way to solve these equations. By iterating g(z) in eq. (7), we can get more accurate approximations. Substituting more accurate approximations of g(z) into $F[b_1(z)]$ will achieve higher orders of approximation of the elimination algorithm. The iterative solution of eqs. (6) and (7) predicts the correct amplitude and phase of all first order internal multiples, with a downward reflection at any reflector.

ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM (1D PRESTACK)

The algorithm in 1D with a normal incident wave points the way to find an algorithm in 1D pre-stack. Consider an example in a 2D world with a 1D earth. In this example, the reflection coefficients and transmission coefficients are both angle dependent. We discussed this example in Zou and Weglein (2014), where we find that the attenuation factors consist of angle dependent transmission coefficients. Following early discussions and work in Ramírez (2007) and Herrera and Weglein (2012), we develop in this paper a complete elimination algorithm for 1D pre-stack data.

Below please find the 1D pre-stack internal-multiple elimination algorithm for acoustic medium (note that the ISS internal-multiple attenuation algorithm is model type independent). Due to the angle dependent reflection coefficients, we can no longer just integrate the data in the k-z domain to get the reflection coefficients as we did in 1D normal incidence, we need to go to the k-q domain where each (k, q) corresponds to one reflection coefficient. The differences between the 1D pre-stack and 1D normal incidence algorithms are (1) the 1D pre-stack algorithm has one more variable k, and (2) use the reflection coefficients in the k-q domain instead of direct integral in the k-z domain.

$$b_3^{IM}(k,2q) = \int_{-\infty}^{\infty} dz e^{2iqz} b_1(k,z) \int_{-\infty}^{z-\varepsilon_1} dz' e^{-2iqz'} F[b_1(k,z')] \int_{z'+\varepsilon_1}^{\infty} dz'' e^{2iqz''} b_1(k,z'') \ ,$$

$$F[b_1(k,z)] = (1/2\pi) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz'dq' \times e^{-iq'z} e^{iq'z'} b_1(k,z')$$
$$/[1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-q'z'''}]^2$$
$$\times [1 - | \int_{z'-\varepsilon}^{z'+\varepsilon} dz'' g(k,z'') e^{iq'z''} |^2] \ ,$$

$$g(k,z) = (1/2\pi) \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dz'dq' \times e^{-iq'z} e^{iq'z'} b_1(k,z')$$
$$/[1 - \int_{-\infty}^{z'-\varepsilon} dz'' b_1(k,z'') e^{iq'z''} \int_{z''-\varepsilon}^{z''+\varepsilon} dz''' g^*(k,z''') e^{-q'z'''}] \ .$$

NUMERICAL TESTS FOR ISS INTERNAL-MULTIPLE ELIMINATION ALGORITHM (1D PRESTACK)

We test the 1D pre-stack acoustic internal multiple elimination algorithm for a two-reflector model. Each layer has a density of 1.0 g/cm^3, 1.2 g/cm^3, 2.0 g/cm^3 and velocity 1500 m/s, 3000 m/s and 4500 m/s, respectively. Fig. 3 shows the data and Figs. 4 and 5 show the attenuation and elimination algorithm predictions, respectively. Fig. 6 to Fig. 13 show different traces in different offsets [the elimination algorithm prediction (red) and attenuation algorithm prediction (green) compared to data (blue)]. We can see the elimination algorithm keeps the correct time and can predict better amplitude.

CONCLUSION

The pre-stack 1D ISS internal multiple elimination algorithm for all first-order internal multiples from all reflectors is proposed in part I of this paper. Numerical tests are carried out to evaluate this new algorithm and to determine the strengths and limitations. The results shows the elimination algorithm can predict improved amplitudes of the internal multiples. In discussing the elimination algorithm, the primaries in the reflection data that enters the algorithm provides that elimination capability, automatically without our requiring the primaries to be identified or in any way separated. The other events in the reflection data, that is, the internal multiples, will not be helpful in this elimination scheme. That is a limitation of this new algorithm. In part II, Zou and Weglein (2014), show that we can replace all b_1 in the elimination algorithm by $b_1 + b_3$ to mitigate this limitation. This algorithm is a part of the three-pronged strategy, that is particularly relevant and provides added-value when primaries and internal multiples are proximal to and/or interfere with each other in either on-shore and off-shore data.

ACKNOWLEDGMENTS

We are grateful to all M-OSRP sponsors for encouragement and support in this research. We would like to thank all our coworkers for their help in reviewing this paper and valuable discussions in this research program. We also thank Dr. James Mayhan for his assist in editing this paper.

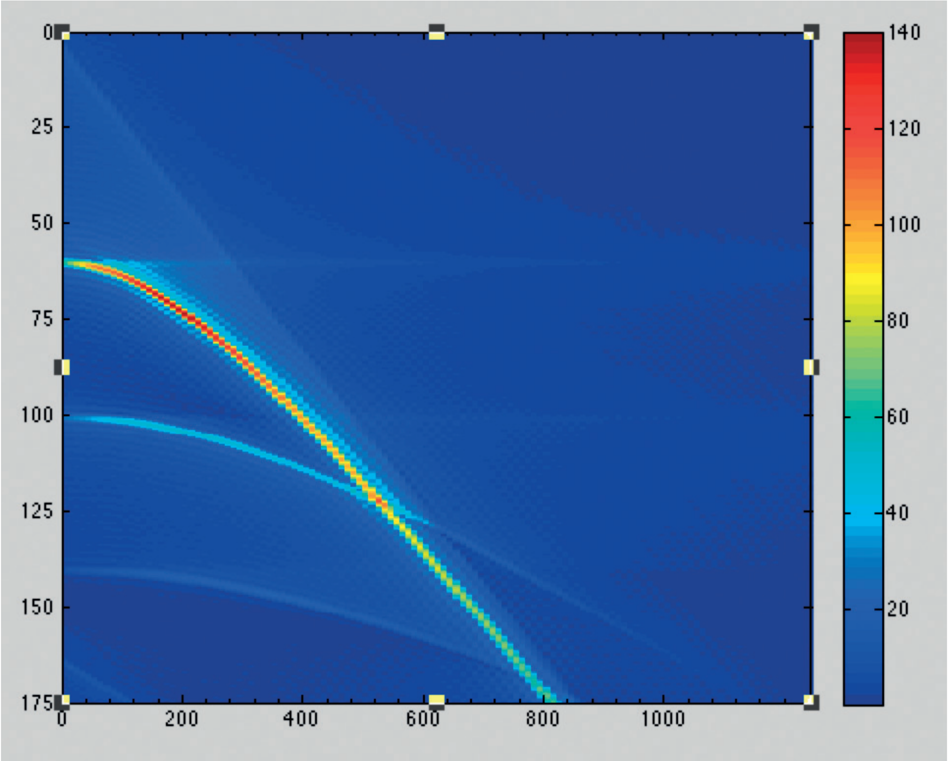

Fig. 3. Data.

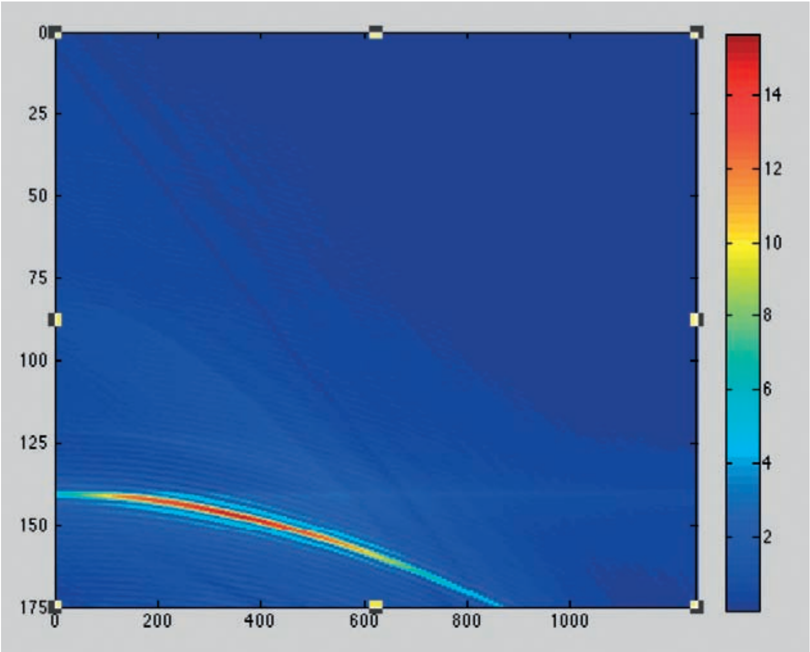

Fig. 4. Internal multiple attenuation prediction.

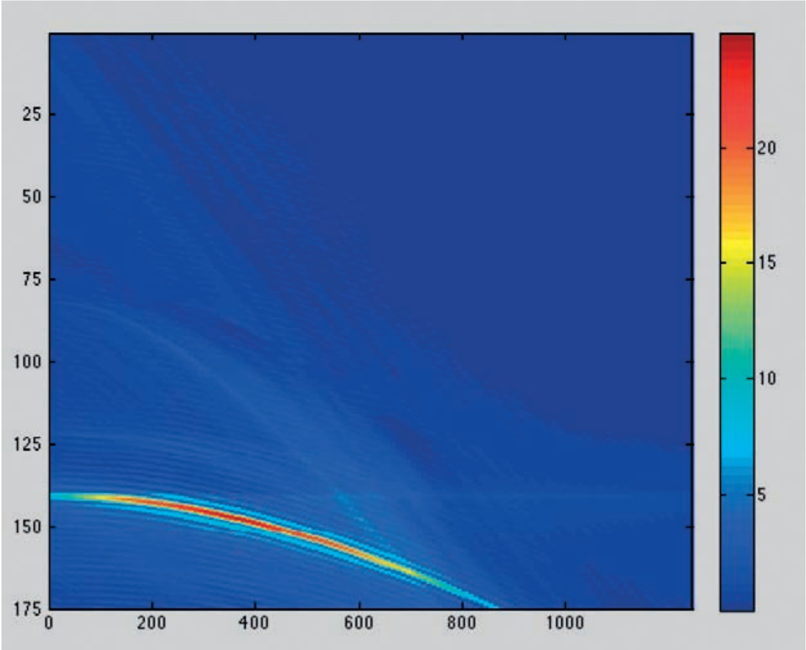

Fig. 5. Internal multiple elimination prediction.

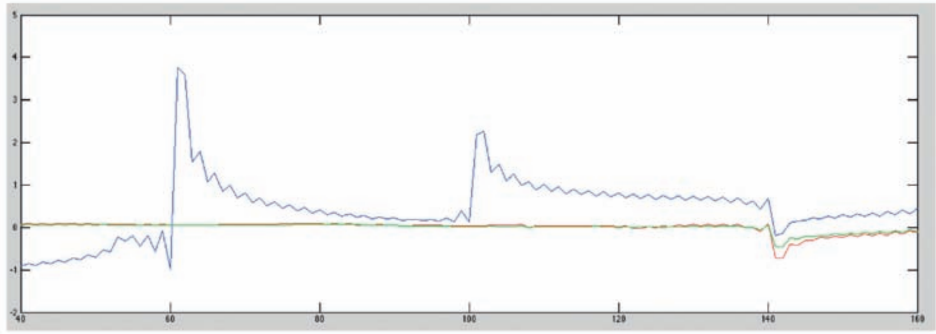

Fig. 6. Attenuation and elimination prediction at offset = 0 m.

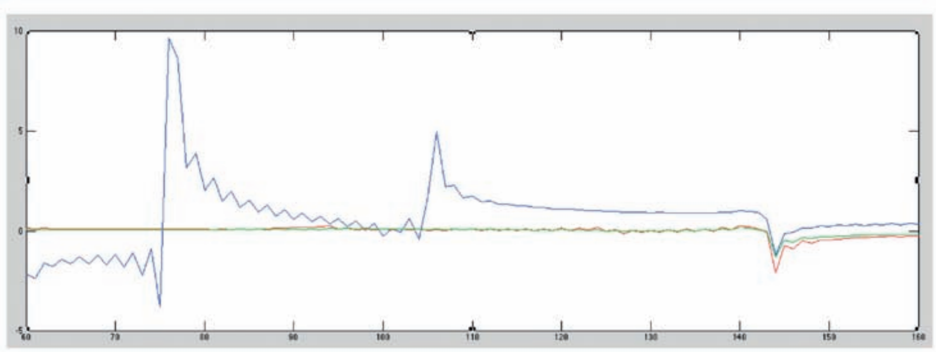

Fig. 7. Attenuation and elimination prediction at offset = 200 m.

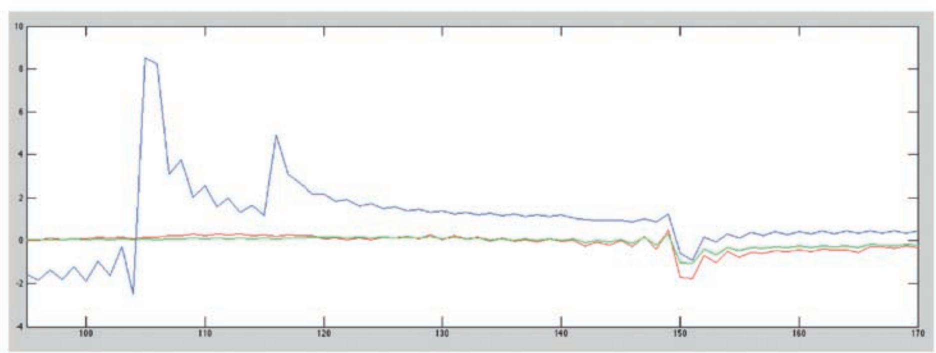

Fig. 8. Attenuation and elimination prediction at offset = 400 m.

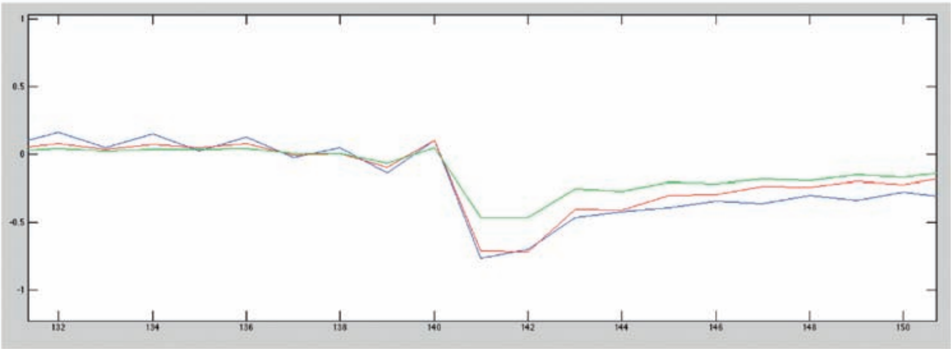

Fig. 9. Prediction at offset = 0 m. After removing the tails of primaries.

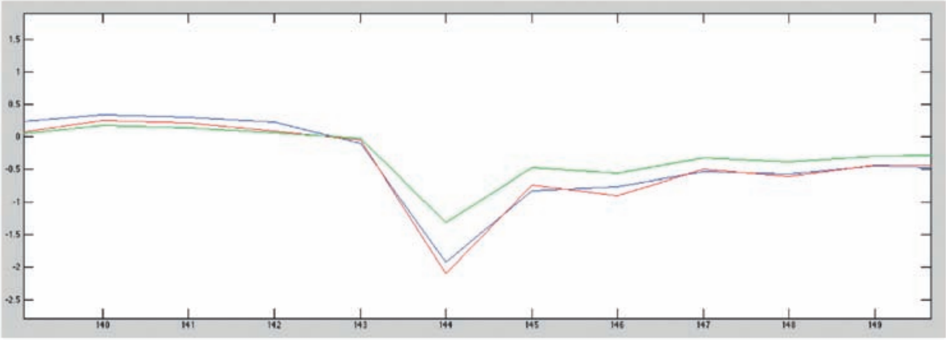

Fig. 10. Prediction at offset = 200 m. After removing the tails of primaries.

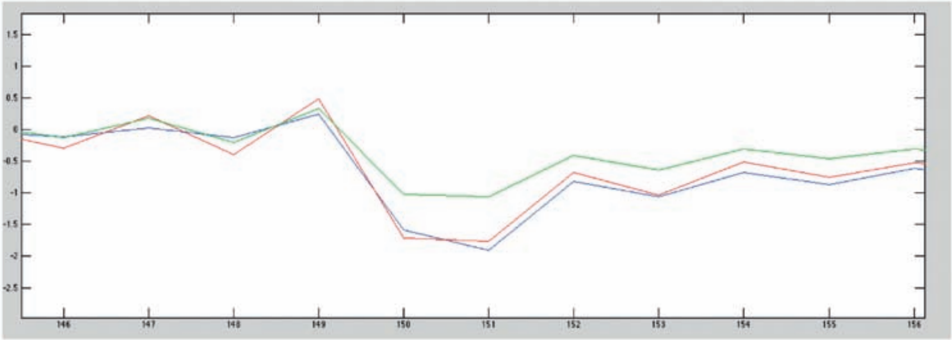

Fig. 11. Prediction at offset = 400 m. After removing the tails of primaries.

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